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Hydrodynamics of Outer Parts of Viscous Decretion Disks of Critically Rotating Stars

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Abstract. Direct centrifugal ejection of matter from a near-critically rotating equatorial surface of hot stars leads to the outflowing viscous decretion disk formation. The mass and angular momentum loss via such disks can significantly influence the evolution of rapidly rotating stars. The viscosity plays a key role in the outward angular momentum transport as well as in the disk thermal energy generation. We study the dynamics of the outer parts of outflowing viscous disks, which is important for the stellar angular momentum loss. We conclude that the outer disk structure is determined by the spatial variations of the disk temperature and alpha viscosity parameter.

1. Basic Theoretical Considerations

The decrease of the moment of inertia due to the stellar evolution may bring the star to the proximity of the critical rotation. A critically rotating star can not spin up any further, consequently the further decrease of the moment of inertia leads to equatorial mass ejection. The net loss of angular momentum is given by

$$\dot{L} = \dot{I}\Omega_{\rm crit}, \tag{1}$$

where $\dot{L}$ is the time rate of change of angular momentum, $\dot{I}$ is the rate of moment of inertia decrease, and the critical rotation frequency is

$$\Omega_{\rm crit} = \sqrt{GM/R_{\rm eq}^3}. \tag{2}$$

The viscosity transports angular momentum to some outer disk radius $R_{\rm out}$, which is typically $R_{\rm out} \gg R_{\rm eq}$. The angular momentum loss from the decretion disk may in this case greatly exceed the angular momentum loss from the stellar wind outflow.

2. Disk Structure

The following hydrodynamic equations describe the structure of the disk integrated over the vertical direction. The mass conservation is (Okazaki 2001; Maeder 2009)

$$R\frac{\partial\Sigma}{\partial t} + \frac{\partial}{\partial R}\left(R\Sigma V_{\rm R}\right) = 0, \tag{3}$$

where Σ is the vertically integrated surface density, $V_{\rm R}$ is the radial component of velocity. The equation of conservation of the radial component of momentum gives

$$\frac{\partial V_{\rm R}}{\partial t} + V_{\rm R}\frac{\partial V_{\rm R}}{\partial R} = \frac{V_\phi^2}{R} - \frac{GM}{R^2} - \frac{1}{\Sigma}\frac{\partial(a^2\Sigma)}{\partial R} + \frac{3}{2}\frac{a^2}{R}, \tag{4}$$

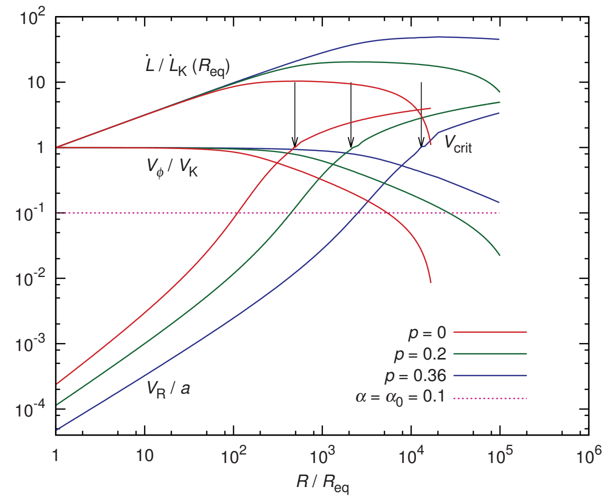

Figure 1. Dependence of relative radial and azimuthal velocities and angular momentum loss rate $\dot{L}/\dot{L}_K(R_{eq})$ on radius for various temperature profiles. $\dot{L}_K(R_{eq})$ is the angular momentum loss rate assuming mass loss directly from the stellar equator. Constant viscosity $\alpha = 0.1$ is considered. Arrows mark the critical radius R_{crit} as a location of a sonic point where the radial velocity $V_R = V_{crit} = a$.

where V_ϕ is the azimuthal component of velocity, a is the speed of sound $a^2 = kT/(\mu m_u)$, μ is the mean molecular weight, m_u is the atomic mass unit. For the conservation of the azimuthal component of momentum we have

$$\frac{\partial V_\phi}{\partial t} + V_R \frac{\partial V_\phi}{\partial R} = -\frac{V_R V_\phi}{R} - \frac{1}{R^2 \Sigma} \frac{\partial}{\partial R}(\alpha a^2 R^2 \Sigma). \quad (5)$$

For the calculations of viscosity we consider here models with power law viscosity decline adopting the α parameter (Shakura & Sunyaev 1973), $\alpha = \tilde{v}_t/a$, where $\tilde{v}_t$ means the average velocity of the turbulent motion of the gas eddies. Here we introduce

$$\alpha = \alpha_0 (R_{eq}/R)^n, \quad (6)$$

where α_0 is the viscosity of the inner region of the disk near the stellar surface and n is a free parameter to describe the radial viscosity decline, $n > 0$. From Eq. 4 in the stationary case follows that at the disk critical point R_{crit}, where the radial velocity equals the speed of sound, one has

$$\frac{V_\phi^2}{R} - \frac{GM}{R^2} + \frac{5}{2}\frac{a^2}{R} - \left.\frac{\partial a^2}{\partial R}\right|_{R_{crit}} = 0. \quad (7)$$

This condition determines the (inner boundary) radial velocity of gas elements at the stellar surface. The viscous torque between adjacent segments of the disk causes viscous dissipation, the temperature of the optically thin outer part of the disk is strongly affected by the irradiation from central star (Lee, Osaki, & Saio 1991). A large portion of radiation will be reflected out of the optically thick part of the disk, consequently the radiative cooling must also be included. For a simplification we assume the temperature profile in a form of the power law

$$T = T_0 (R_{eq}/R)^p, \quad (8)$$

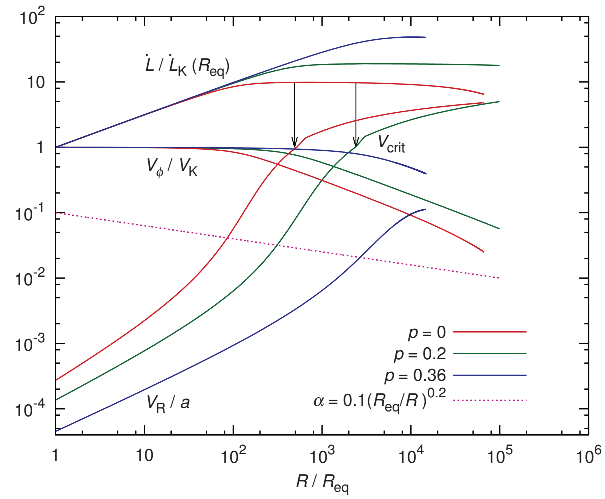

Figure 2. Same as Fig. 1 with variable α parameter. The inner boundary viscosity α_0 is set 0.1, the viscosity profile is estimated here as a radial power law $\alpha \sim R^{-0.2}$.

where T_0 is the temperature of the disk near the stellar surface and p is a free parameter ($p < 1$) with estimated values in range between 0 and 0.4. The temperature distribution in the inner region of the disk is nearly isothermal ($T_0 = \frac{1}{2}T_{\rm eff},\ p = 0$, Carciofi & Bjorkman 2008), but for the calculations of the structure of outer part of the disk it is reasonable to consider also the power law temperature decline.

3. Numerical Approach

We solve the system of hydrodynamic equations in cylindrical coordinates including the mass conservation equation (Eq. 3) and the R and ϕ components of the equation of momentum conservation (Eqs. 4 and 5) supplemented by appropriate boundary conditions (Okazaki 2001; Krtička, Owocki, & Meynet 2011) assuming a stationary flow. We selected the star with parameters corresponding to the spectral type B0 (Harmanec 1988), the effective temperature $T_{\rm eff} = 30\,000$ K, mass $M = 14.5\,\mathrm{M}_\odot$, and radius $R = 5.8\,\mathrm{R}_\odot$. For the numerical differentiation at selected radial grid we use the Newton-Raphson method.

4. Models

We examine behaviour of radial and azimuthal velocity and the angular momentum loss in the disk with various temperature and α viscosity profiles. In case of constant viscosity (see Fig. 1) the disk rotates in an inner region with Keplerian velocity $V_K(R)$, in a supersonic region its azimuthal velocity as well as the angular momentum loss rate rapidly decrease, and at large radii these quantities may become in this model even negative.

We present here two models where the viscosity profile is estimated as a radial power law. From the model with $\alpha \sim R^{-0.2}$ (see Fig. 2) follows that for lower values of parameter p in temperature profile the values of angular momentum loss at large radii

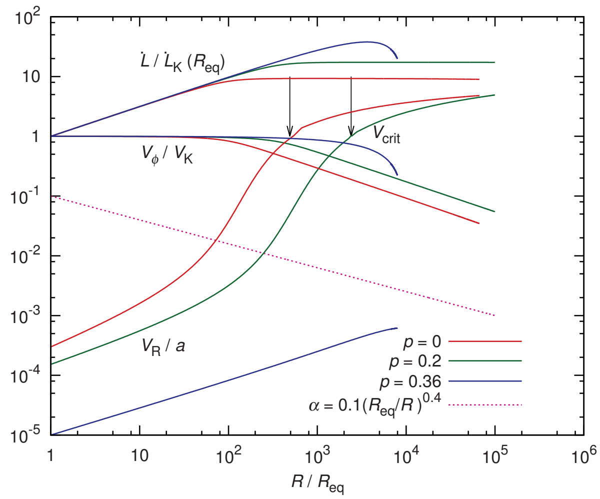

Figure 3. Same as Fig. 1, the inner boundary value of viscosity α_0 is set 0.1, viscosity profile behaves like $\alpha \sim R^{-n}$, where $n = 0.4$.

remain constant, this implies that the decrease of azimuthal velocity at large radii obeys R^{-1}. For higher values of parameter p in temperature profile the radial velocity may not reach the critical velocity V_{crit}, at large radii may azimuthal velocity and angular momentum loss rate rapidly decrease. Fig. 3 shows another model with $\alpha \sim R^{-0.4}$ where the profiles are similar as in the previous case but with stronger radial dependence, for lower values of parameter p the angular momentum loss in supersonic region remains constant up to very large radii, the azimuthal velocity profile consequently obeys R^{-1}. For higher values of parameter p the radial velocity profile remains deeply subsonic.

5. Conclusions

The model with constant viscosity shows the unphysical decrease of angular momentum loss at large radii. As a solution of this problem we introduce the models with power law viscosity decline, up to certain value of p parameter in temperature profile the models show constant angular momentum loss in supersonic region.

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