

## Mass Determination of Small Solar System Bodies with Ground-based Observations

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**Abstract.** Asteroid masses are the key values for physical studies of their internal structure and also for the dynamics of the asteroid families. Dynamical method is based on the analysis of the perturbing effects of a bigger mass asteroid on the orbit of a smaller mass body (another asteroid, spacecraft, asteroid satellite). The use of recent ground-based observations by the CCD astrometry, reduced using contemporary astrometric catalogues, makes it possible to improve the previously adopted masses and to determine new ones. The present study is based on the observations made at Nikolaev Observatory, at TÜBİTAK National Observatory, and observations taken from the Minor Planet Center database. Integration of relativistic equations of motion was made with the initial conditions, taken from the DE405 and JPL HORIZONS ephemerides. The newly determined asteroid masses are compared with recent determinations made by other authors.

### 1. Introduction

Asteroids masses under discussion are determined on the whole by dynamical method, based on the analysis of perturbations of “zero-mass” asteroids (the name well depicts the assumption used) which are gravitationally perturbed by the massive ones.

The principal difficulty of the problem is in the selection of observations to be used for fitting the parameters of the dynamical model. The past observations of selected asteroids are few, heterogeneous, and are suspected of having systematic errors of various nature (different catalogues of reduction, errors in the times of observation, instrumental errors present, etc.). All these can lead not only to increasing the dispersion, but also to biasing the mass estimates. In some cases calculated masses come out to be negative (Vasiliev & Yagudina 1999), meaning that observations used are not accurate enough for such studies.

The above mentioned difficulties can, to some extent, be overcome by means of the recent positional observations. Number of asteroid observations, accessible through the Minor Planet Center<sup>1</sup>, are growing exponentially; for some selected asteroids, the number of observations in the 1999-2005 period increased

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<sup>1</sup><http://cfa-www.harvard.edu/cfa/ps/mpc.html>

by a factor of two in comparison with the past. Besides, present accurate and dense catalogues, such as UCAC2 and CMC14, provide an opportunity to solve astrometric problems at the level of  $0.05''$  (Stone 2000). These are encouraging arguments for reliable asteroid mass determinations.

## 2. Observations and Some Results

High positional accuracy of observations and long observational period of asteroids are necessary and important for the success of dynamical mass determination. The observations sent in to the Minor Planet Center are results of different observational programs and, therefore, the size of the observational sample for a given asteroid can be a matter of chance. Therefore our principal aim is to make observations of specially selected asteroids which have or will have perturbations greater than 50 mas in any of the observed coordinates. The conditions of selected encounters are listed in Fienga et al. (2003), Galad (2001), Galad & Gray (2002), and will not be repeated here.

The Russian-Turkish Telescope, RTT150<sup>2</sup> ( $D = 1.5m$ ,  $F = 11.6m$ ), at the TÜBİTAK National Observatory (Turkey), equipped with contemporary CCD of ANDOR ( $2K \times 2K$ ,  $13.5 \times 13.5 \mu m^2$ ), proved to have good qualities for astrometric observations. Field of view is  $8' \times 8'$ .

The observations of selected asteroids of 11 to 18 magnitudes were started in May 2004 at the RTT150 for testing the positional accuracy within the joint project between the TÜBİTAK National Observatory (Turkey), Kazan State University (Russia), and Nikolaev Astronomical Observatory (Ukraine) (Aslan et al. 2005). About 4 thousand CCD images of 58 asteroids were obtained by the end of 2005, of which about 700 were discarded because of bad quality or because of a small number of reference stars. Standard errors of a single position derived from these observations are plotted in Figures 1 and 2 as a function of magnitude. Almost all the internal observational errors (dots) for the list of selected asteroids up to magnitude 17 are within  $0.1''$ , with a mean less than  $0.05''$  in both coordinates, and do not depend on the magnitudes of the objects. The slope of the slight trend in Figure 2 is not significantly different from zero.

External uncertainties of observations were determined from the (O-C) values of objects with no less than 3 epochs of observations (“O” was calculated using the UCAC2 catalogue, “C” was calculated with the “HORIZONS” system<sup>3</sup>). There were 10 such asteroids within the entire period of observation. The values of these uncertainties are shown as crosses in Figures 1 and 2. The mean values of the external uncertainties are somewhat greater than the internal errors in right ascension and reach  $0.1''$ . The internal and external uncertainties in declination are about  $0.05''$ . The fact that the external uncertainty of observations of selected asteroids in right ascension is more than the internal one by a factor of two needs explanation.

Mean standard errors of asteroid positions in right ascension and declination obtained in 5 observatories in 2004-2005 are quoted in the Table 1. The data

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<sup>2</sup><http://www.tug.tubitak.gov.tr/rtt150>

<sup>3</sup><http://ssd.jpl.nasa.gov/?horizons>

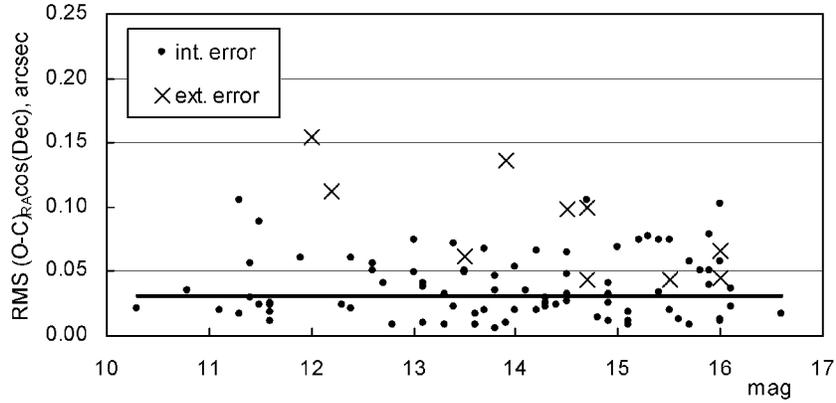


Figure 1. Standard errors of a single (O-C) in right ascension as a function of magnitude. Dots are internal errors of observations for selected asteroids, crosses are external errors.

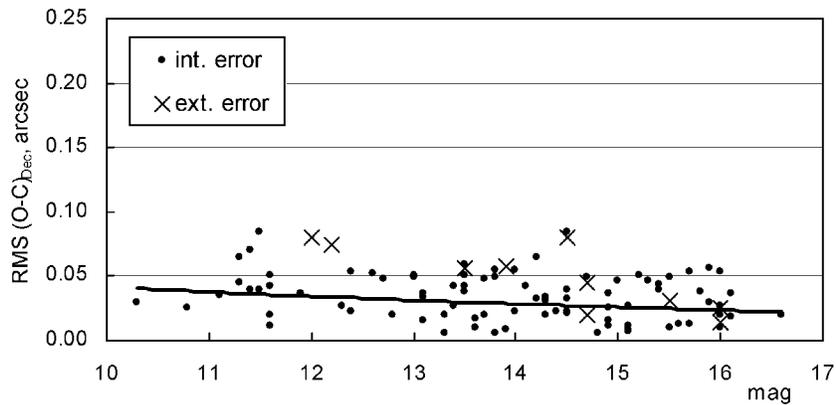


Figure 2. Standard errors in declination, as in Figure 1.

are taken from the site of the Minor Planet Center<sup>4</sup>, where these observations are marked with suffix ‘h’, meaning ‘high precision astrometry’. In the last row of the table, the results from the current joint project at the RTT150 are given.

As one can see from the figures and the table above, the accuracy of asteroid positions from the RTT150 is at the level of best achievements in this field, and the positions are good enough for use in mass determinations of large asteroids.

<sup>4</sup><http://cfa-www.harvard.edu/iau/special/residuals.txt>

Table 1. External standard errors, in arcseconds, of position from asteroid observations at various observatories in 2004-2005.

Observatory code and name	RMS in right ascension	RMS in declination	Number of observations
(089) Nikolaev Observatory	0.21	0.18	250
(413) Siding Spring Observatory	0.14	0.15	1485
(568) Mauna Kea	0.18	0.20	6570
(673) Table Mountain Observatory, Wrightwood	0.01	0.07	3600
(689) U.S. Naval Observatory, Flagstaff	0.17	0.19	122666
Joint project - RTT150	0.10	0.07	3300

### 3. Dynamical Method of Asteroid Mass Determination and Preliminary Results

Motion of an asteroid in barycentric coordinates of the Solar System can be well modelled with the relativistic equation of motion for particle bodies in the isotropic, Parameterized Post-Newtonian (PPN) n-body metric (Newhall et al. 1983). We use the same equation for point-mass interactions as was used in DE102, with the exception of “asteroids” item therein.

As the results in mass determinations should be in agreement with the accepted system of constants, used in the latest standard DE405 theory, we decided not to integrate explicitly equations of motions for the Sun and major planets, but to use their positions and velocities directly from the DE405. The numerical integration of the equations of motion was carried out using a variable step-size, variable order Adams-Moulton method (the maximum order is 12).

For test purposes, we have chosen several pairs of asteroids which had given negative mass values from calculations made earlier (Vasiliev & Yagudina 1999). The data presented in Table 2 give close encounters, used for the calculations, and the numbers of observations of perturbed asteroids, which can be accessed through Minor Planet Center. As one can see from the table, the numbers of observations increased several times in recent years.

Method of improvement of initial mass values is not new. It is based on the assumption of “good” initial conditions, masses, and comparatively small corrections to be applied. Initial conditions of asteroids (positions and velocities) were taken from the HORIZONS system. The necessary numerical requirement of positivity for mass value, introduced by linear inequality into the least squares, distinguishes our determinations from those made earlier.

The necessary corrections for light-time aberration and diurnal parallax were applied to the geometrical position of perturbed asteroid to get the topocentric astrometric position. These positions can be directly compared with spherical coordinates of the asteroid, obtained from the measurements in the system of reference catalogue.

Table 2. Circumstances of encounters and numbers of observations of perturbed asteroids.

Perturbing asteroid	Perturbed asteroid	Date of close encounter	Number of available observations		Standard error of present fit, arcsec
			before 2000	before 2006	
(7) Iris	(836) Jole	Feb 1989	136	486	0.68
(10) Hygiea	(3946) Shor	Apr 1998	135	777	0.59
(24) Themis	(2169) Taiwan	Dec 1974	193	729	0.62
(45) Eugenia	(2560) Sieigma	May 1968	194	712	0.66
(45) Eugenia	(2814) Vieira	Nov 1983	198	615	0.68
(45) Eugenia	(308) Polyxo	Nov 1985	697	1107	0.71
(52) Europa	(3019) Kulin	Nov 1988	254	851	0.66
(52) Europa	(1558) Jarnefelt	Jul 1990	182	551	0.67
(87) Sylvia	(1081) Reseda	Aug 1964	243	680	0.66
(165) Loreley	(1913) Sekanina	Jul 1981	208	716	0.66

Table 3 gives a summary of masses from the close encounters in comparison with the earlier determinations. The uncertainties, given here, are one-sigma uncertainties.

Table 3. Comparison with the recent mass determinations of 7 asteroids, masses in  $10^{-10}M_{\odot}$ 

Asteroid	Vasiliev & Yagudina, 1999	Michalak, 2001	Kochetova, 2004	Present calculations, 2006
(7) Iris	$-0.40 \pm 0.35$	no value	$0.120 \pm 0.009$	$0.09 \pm 0.01$
(10) Hygiea	$-0.45 \pm 0.76$	$0.735 \pm 0.048$	$0.406 \pm 0.019$	$0.21 \pm 0.03$
(24) Themis	$-0.18 \pm 0.34$	no value	no value	$0.01 \pm 0.02$
(45) Eugenia	$-0.43 \pm 0.14$	no value	no value	$0.01 \pm 0.03$
(45) Eugenia	$-0.39 \pm 0.22$			
(45) Eugenia	$-0.09 \pm 0.09$			
(52) Europa	$-0.57 \pm 0.20$	$0.400 \pm 0.078$	$0.127 \pm 0.025$	$0.36 \pm 0.04$
(52) Europa	$-1.03 \pm 0.77$			
(87) Sylvia	$-0.22 \pm 0.10$	no value	no value	$0.18 \pm 0.09$
(165) Loreley	$-1.39 \pm 0.46$	no value	no value	$0.16 \pm 0.10$

Results of the present calculations can not easily be compared with the previous determinations due to the somewhat different ideas, different datasets, and different weighting schemes used. All the discordant values may be due to systematic biases in the previous determinations and/or a consequence of past encounters and infrequent observations in the past with non-evident systematic

errors. Here lies the need for special and accurate observations. The observations at the RTT-150 satisfy these needs and are very useful for dynamical determinations of masses.

#### 4. Conclusions

1. The first results of our regular observations carried out at the RTT150 have shown the positional accuracy to be high, with a mean internal uncertainty  $\sim 50$  mas of a single position of an asteroid up to magnitude 17. The achieved accuracy allows us to expect successful solution of the problem of asteroid mass determination within our joint international project and about the use of this telescope for ground-based support of astrometry part in future space mission GAIA.
2. Preliminary masses of 7 asteroids were obtained via the dynamical method using the ground-based optical observations. We used those encounters where earlier mass determinations had given negative values (Vasiliev & Yagudina 1999). Our determinations are distinct as they are based on a larger number of observations with the set requirement of mass positivity.
3. Reliable mass determinations are expected under the conditions of special and accurate observations made at the RTT150.

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